

## **Experimental Comparison of Trajectory Planning Algorithms for Wheeled Mobile Robots**

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**ABSTRACT :** *In this paper, we present an experimental approach to compare various trajectory planning methods for practical application of wheeled mobile robots. The first method generates a trajectory according to the acceleration limits of the mobile robot and its relationship with the curvature of the planned path. The second method is an improvement of the conventional convolution-based trajectory generation method, on which the heading angles of a curved path is being considered. Due to the limited scope of the considered constraints of the previous approaches, A third approach that conserves the merits of the convolution operator is proposed to consider the high curvature turning points of a sophisticated curve such as a Lemniscate of Gerono, which causes geometrical limitations during robot navigation. All methods are compared experimentally on a two-wheeled mobile robot. The goal of the experiment is to determine which approach meets the criteria of time optimality and sampling time uniformity while considering the physical limits of the mobile robot and the geometrical constraints of the planned path.*

**Keywords** - *Wheeled Mobile Robots, Trajectory Planning, Physical Limits, Convolution, Lemniscate of Gerono*

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### **I. INTRODUCTION**

Mobile robotics is an active research area that is continuously evolving to enable mobile robots display precise movements through simple user-generated velocity commands. Nowadays, unmanned mobile robot application domains has extended in industrial as well as in service sectors [1]. It has also attracted lot of interests from scientific, governmental, and residential areas [2-3]. Another great result from this advancement is that places unreachable to mankind before are now accessible through meticulous operations of mobile robot systems such as exploration of outer space bodies [4], recognition of targets in conflict and disaster zones [5], navigation in hazardous environments, and dangerous military operations [6]. In cases where the operating environment is known, navigation of mobile robots is a systematic operation that is divided into three main phases; path planning [7], trajectory generation [8], and tracking control [9].

Trajectory generation produces the required velocity commands for the robot to transverse the planned path as a function of time. To obtain feasible trajectories, movement of the mobile robot could be constraint depending on its physical limit such as maximum velocity [7] and acceleration limits [10-11] and maximum jerk for accurate control along higher-polynomial planned paths [12]. In a curved path, geometrical constraints relating to high curvature turning points could exist which is non-negligible. In this event, rigorous robot control and operation could encounter issues such as inability to reach the desired target point or slowing down-not reaching the expected terminal velocity. Geometrical constraint is an inevitable problem to be solved because existence of obstacles in practical robot navigation is imminent.

Trajectory planning methods that considers the acceleration limits of the mobile robot is given by researchers in [10-11]. A trajectory generation approach based on convolution is presented in [12]. However, the conventional convolution operator does not consider the heading angles along a path. Convolution-based trajectory generation for a curved path is achieved through the improvement found in [8]. Yet, it was not able to generate feasible trajectory that enables the mobile robot to track a planned path with high curvature.

The contribution of this paper is divided into two principal parts. First, a solution to the drawbacks of previous researches is presented. Second, all the denoted trajectory planning algorithms and the proposed method were given experimental overviews and was compared in the criteria of time optimality, sampling time uniformity, and ability to consider both the physical constraints of the mobile robot and the geometrical constraints of the planned path.

### **II. TRAJECTORY PLANNING ALGORITHMS**

#### **2.1 Considering Acceleration Limits**

Velocity command generation that considers the acceleration limits of the mobile robot is presented by researchers in [9] and [10]. If the acceleration of the robot exceeds the limitation according to the robot's specifications, slipping or skidding occurs. The maximum allowable central driving velocity of the mobile robot

is restricted by the curvature of the path and its relationship with the radial acceleration,  $a_r$ , of the mobile robot defined as:

$$v(u(t)) = \sqrt{a_r \rho(u(t))} \tag{1}$$

here the radius of curvature,  $\kappa$ , is defined as  $\rho = 1/\kappa$ , where  $\rho$  is radius of the path.

When the radial acceleration limit is satisfied at each point along the curve, the corresponding velocity is realizable on that point. The velocity of the robot is determined from the initial point, terminal point, and in present, from the turning points along the curve. The points with the highest local curvature values are considered the turning points along the curve. Velocity are calculated assuming that the radial acceleration is at maximum value on that point. From the starting points, the velocity at the next point is evaluated assuming that the robot is accelerated uniformly between each sampling points [9]. The maximum allowable velocity is the lowest value at each sampling point of the combined velocity profiles.

**2.2 Convolution-based trajectory planning**

A study stated that a smooth trajectory for a mobile robot is possible to be generated using convolution. Assuming that an impulse function is a convoluted rectangular function with a unit area in certain time duration and an arbitrary input function derived from the distance between two points are given, the output function is the required velocity for the robot to travel the given input distance. Thus, successive convolution operations by applying the resulting output as the input of the system would produce smoother trajectories [12]. A recursive form of the convolution sum is expressed as:

$$v_a(t) = y_n[k] = \frac{y_{n-1}[k] - y_{n-1}[k - m_n]}{m_n} + y_n[k - 1] \tag{2}$$

$$k = t / T_s, m_n = t_n / T_s$$

where  $t$  denotes the time duration with respect to the actual travel distance along the curved path,  $T_s$  represents the sampling time, and  $n$  is the number of repetitions the convolution operator is executed—determined from the degree of the curved path.

Although the convolution operator was able to display wider scope of physical limits that it can consider, it can only be applied to the linear distance between two points. Meaning, the actual distance along a curved path was not considered. Hence, an improvement was presented in [8], where the actual distance along a curved path was computed. The heading angles at each point of the curve was calculated by modifying the defining parameter of the curve as a function of time.

**2.3 Proposed method**

Aside from experimental contrast of prior trajectory generation algorithms, this paper presents a solution to the drawbacks that were found during experimentation. Since the convolution operator has an advantage in satisfying most possible physical constraints of the mobile robot during trajectory planning, these merits were conserved. In order for the convolution-based method to consider the geometrical constraints of a curved path, the curvature of the path was evaluated using equation. The turning points are identified with its corresponding curvature values. On each turning point, the velocity is computed using (1). The proposed velocity profile is generated by application of the recursive form of the convolution sum in (2) to the starting point of the curve. The backward velocity from the terminal point is neglected for it would result the same velocity profile as the initial point.

For the turning points, the first portion is evaluated by convolution in the forward direction from the turning point to the terminal point. Meanwhile, the second portion is generated in the backward direction towards the initial point. As a result, all generated velocity profiles are combined and sorted to find the lowest velocity at each point which determines the proposed velocity command generation. Due to the limited velocity at the turning points, the travelling time is increased to conform the total travel distance by:

$$dt = \frac{dS(u)}{v(u)} \tag{3}$$

where  $dS$  denotes the distance between each sampling point and  $dt$  the corresponding sampling time.

As a consequence, non-uniform sampling time is attained through the alteration. Hence, a variation of linear interpolation is conducted to acquire uniformity:

$$v_{uni}(t) = v_c(t_{k-1}) + \frac{v_c(t_k) - v_c(t_{k-1})}{t_k - t_{k-1}} \times (t - t_{k-1}) \tag{4}$$

where  $v_{uni}$  is the proposed velocity in uniform sampling time and  $t$  is the arbitrary uniform sampling time, bounded within  $t_{k-1} \leq t \leq t_k$ .

### III. EXPERIMENTAL RESULTS

The established algorithms, together with the proposed approach were implemented to a differential drive two-wheeled mobile robot with actuators separated by approximately 0.4 m. The mobile robot was constraint at 0.9 m/s maximum velocity, 2.0 m/s<sup>2</sup> maximum acceleration, and 1.0 m/s<sup>3</sup> maximum jerk. The sampling time was set to 0.02 s for each point. The initial and terminal velocities were configured to 0 m/s, which are the typical values in practical navigation. The planned path was designed using a more sophisticated curve, Lemniscate of Geron, which is defined by the parametric equation:

$$x(u) = x_0 + r \sin(u) \tag{5}$$

$$y(u) = y_0 + r \sin(2u)$$

where  $\pi/12 \leq u \leq 11\pi/6$ ,  $(x_0, y_0)$  and r denotes the origin and radius of the lemniscate, respectively.

The curve was chosen for it displays sharp turning points that contemplates high curvature which is suited for this experiment. The velocity for the first method is shown in Figure 1. Although the turning points of the curve was considered, only the acceleration limits were satisfied. Moreover, the sampling time at each point along the curve was found to be non-uniform, which would lead to issues regarding navigation control especially in real-time operation. Also, inaccuracy issues regarding total travel time computation occurs when either the initial or terminal velocity is zero-value.

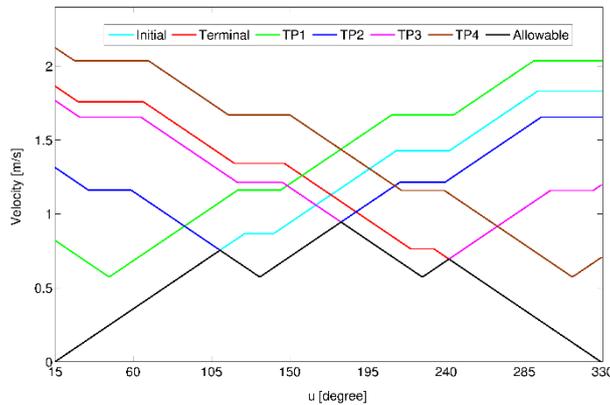


Figure 1: Velocity profile considering acceleration limits

Figure 2 shows the generated central velocity profiles based on the convolution operator. Figure 2(a) displays the result from the prior method that considers the heading angles of a curved path. On the other hand, Figure 2(b) shows the proposed method that enhances the former to consider the high curvature turning points along the planned path.

The total travel time calculated from both convolution methods are 8.9s and 11.2 s, respectively. The previous method is drastically faster than the proposed due to the fact that it does not consider the curvature of the path. Due to the velocity limits from the turning point, the mobile robot slows down when approaching the turning points of the curve.

Figure 3 shows the generated trajectory using both velocity profiles from Figure 2. As shown in Figure 3(a) the convolution method that does not consider the curvature of the path that causes geometrical constraints was not able to follow a sophisticated path like the lemniscate of Geron for it has sharp turning points. Meanwhile the proposed method was able to track down the reference path accurately as shown in Figure 3(b). Thus, a slight increase of travel time for better accuracy is presented by the previous method.

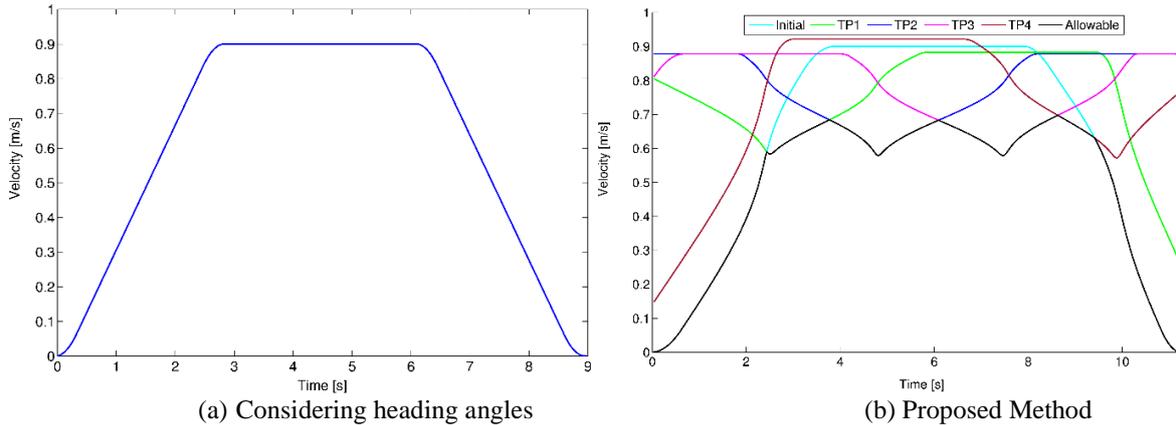


Figure 2. Convolution-based velocity profiles

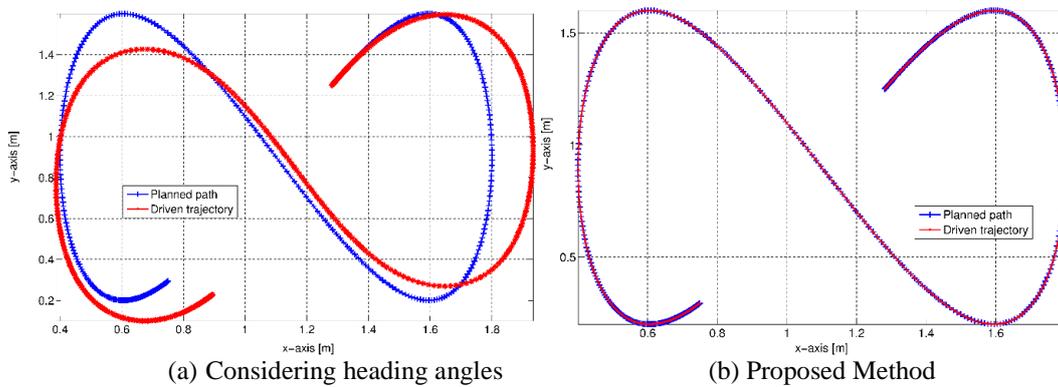


Figure 3. Trajectories driven by velocity profiles

#### IV. CONCLUSION

An experimental comparison of three different trajectory generation methods for actual navigation of a mobile robot is presented. Experiments have shown that established researches have drawbacks in tracking a path with high curvature. The first method that considers the acceleration limits of the mobile robot produces a trajectory with non-uniform sampling time and has issues when the initial or terminal velocity is zero-value. The convolution-based method that considers the heading angles of a curved path was able to consider wider scope of physical limits of the mobile robot, the geometrical constraints were not satisfied which results to inability to track the planned path.

Therefore, a trajectory generation method is recommended to enable a mobile robot to track a high curvature path while considering the physical limits of the mobile robot in uniform sampling time and fast travel time.

Future work will include usage of tracking control systems and other advanced methods, like adaptive trajectory planning and obstacle avoidance. In addition, implementation in a real-time operating system will be developed.

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